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A Novel Approach to the Design of Unknown Input Observers

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Abstract

A novel state estimator design scheme for linear dynamical systems driven by partially unknown inputs is presented. It is assumed that there is absolutely no information available about the unknown inputs, and thus unlike a number of past studies, no apriori assumption is made about the nature of these inputs. A simple but yet elegant approach for designing a reduced order unknown input observer (UIO), with pole placement capability is proposed.

1) Introduction

The problem of estimating the state of a linear time invariant dynamical systems driven by both known and unknown inputs has been the subject of a number of research studies in nearly the past two decades (e.g. see [1-8]). The problem is of considerable importance since in practice there are many situations where there are plant disturbances present, or some of the inputs to the system are inaccessible, and therefore a conventional observer which assumes the knowledge of all inputs can not be used. An observer capable of estimating the state of a linear system with unknown inputs can also be of tremendous use when dealing with the problem of instrument fault detection, isolation, and accommodation (FDIA), since in such systems most actuator failures can be generally modeled as unknown inputs to the system [9-10].

Basically there have been two approaches to the problem of designing UIOs. A number of these attempts such as [1-2], assume some apriori information about the unmeasurable inputs. Specifically Johnson [1] assumes a polynomial approximation to these inputs, and in [2] it is assumed that the unknown inputs can be modeled as the response of a known dynamical system represented by a constant coefficient differential equation. The next category of UIO studies assumes no knowledge of the inaccessible inputs [3-8]. Among the earlier works is that of [3], which proposes a simple observer that is capable of reconstructing the entire state of a linear system with the presence of the unknown inputs. However, no systematic guideline for designing such an observer was given in [3]. Since then several authors have provided various techniques for designing such observers. In [4] the concept of multivariable systems inverse is used, whereas [5] provides a necessary condition for existence of the UIO, and a design based on generalized inverse of matrices. Observers with a similar structure to that of [5] but simpler techniques were proposed by [6], and [7]. Finally, [8] provides a necessary and sufficient condition for the design of such UIOs.

The UIO design considered in this paper falls under the second category of approaches described above. That is, no knowledge about the nature of the unknown inputs is assumed. In the next section a very simple and straight forward procedure for designing a reduced order UIO with pole placement capability is presented.

2) Design and Structure of the UIO

Consider a linear time invariant dynamical system represented in the following state space formulation

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} + \mathbf{DV} \tag{1}$$

$$\mathbf{Y} = \mathbf{CX} = [\mathbf{0} \quad \mathbf{I}]\mathbf{X} \tag{2}$$

where $\mathbf{X} \in \Re^n$, $\mathbf{U} \in \Re^q$, $\mathbf{V} \in \Re^m$, and $\mathbf{y} \in \Re^p$ are the state, known input, unknown input, and the output of the system, respectively. Notice from (2) that a special structure for the measurement matrix **C** is assumed. This is not a restrictive assumption since as long as **C** is a full rank matrix there will always exist a similarity transformation that if performed on a given system will result in the desired output equation [11].

In the following development a similar condition to that given by [5], and used by others such as [8] is used for designing the UIO. The existence condition in [5] states that for designing a stable observer it is necessary that

$$rank(\mathbf{CD}) = m \qquad with \quad m \le p \tag{3}$$

In this study ***we will require that the number of the unknown inputs be less than the number of outputs, that is, m<p.***

Using the above information, the dynamical system described in (1) and (2) can be written in a partitioned form as

$$\dot{\mathbf{X}} = \begin{pmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \mathbf{A}_3 \end{pmatrix}\mathbf{X} + \begin{pmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \\ \mathbf{B}_3 \end{pmatrix}\mathbf{U} + \begin{pmatrix} \mathbf{D}_1 \\ \mathbf{D}_2 \\ \mathbf{D}_3 \end{pmatrix}\mathbf{V} \tag{4}$$

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I}_{(p-m)x(p-m)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}_{mxm} \end{bmatrix}\mathbf{X} \tag{5}$$

where the matrices $\mathbf{A}_i$, $\mathbf{B}_i$, and $\mathbf{D}_i$ all have appropriate dimensions, and the state vector **X** is partitioned as

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{pmatrix} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix}$$

and $\mathbf{x}_1 \in \Re^{n-p}$ is the vector whose estimate is required.

From the structure of observation matrix $\mathbf{C}$, and the necessity condition assumed above, it is easy to show that

$$rank\begin{pmatrix} \mathbf{D}_2 \\ \mathbf{D}_3 \end{pmatrix} = m \tag{6}$$

UIO Design

From (6), without loss of any generality, it can be assumed that $\mathbf{D}_3$ is nonsingular. Therefore, the following matrix operator can be defined

$$\mathbf{T} = \begin{pmatrix} \mathbf{I} & \mathbf{0} & -\mathbf{D}_1\mathbf{D}_3^{-1} \\ \mathbf{0} & \mathbf{I} & -\mathbf{D}_2\mathbf{D}_3^{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{pmatrix} \tag{7}$$

Post multiplying equation (4) with (7) results in

$$\begin{pmatrix} \dot{\mathbf{x}}_1 - \mathbf{D}_1\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 \\ \dot{\mathbf{y}}_1 - \mathbf{D}_2\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 \\ \dot{\mathbf{y}}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 - \mathbf{D}_1\mathbf{D}_3^{-1}\mathbf{A}_3 \\ \mathbf{A}_2 - \mathbf{D}_2\mathbf{D}_3^{-1}\mathbf{A}_3 \\ \mathbf{A}_3 \end{pmatrix}\mathbf{X} + \begin{pmatrix} \mathbf{B}_1 - \mathbf{D}_1\mathbf{D}_3^{-1}\mathbf{B}_3 \\ \mathbf{B}_2 - \mathbf{D}_2\mathbf{D}_3^{-1}\mathbf{B}_3 \\ \mathbf{B}_3 \end{pmatrix}\mathbf{U} + \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{D}_3 \end{pmatrix}\mathbf{V} \tag{8}$$

Notice that in equation (8) the unknown inputs enter only through the third row, and the first two rows are independent of any unknown inputs. Thus, defining

$$\overline{\mathbf{A}}_i = \mathbf{A}_i - \mathbf{D}_i\mathbf{D}_3^{-1}\mathbf{A}_3$$

$$\overline{\mathbf{B}}_i = \mathbf{B}_i - \mathbf{D}_i\mathbf{D}_3^{-1}\mathbf{B}_3$$

The first two rows of (8) can be written as

$$\dot{\mathbf{x}}_1 - \mathbf{D}_1\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 = \overline{\mathbf{A}}_1\mathbf{X} + \overline{\mathbf{B}}_1\mathbf{U} \tag{9}$$

$$\dot{\mathbf{y}}_1 - \mathbf{D}_2\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 = \overline{\mathbf{A}}_2\mathbf{X} + \overline{\mathbf{B}}_2\mathbf{U} \tag{10}$$

Partitioning $\overline{\mathbf{A}}_i$ as

$$\overline{\mathbf{A}}_i = [\overline{\mathbf{A}}_{i1} \quad \overline{\mathbf{A}}_{i2} \quad \overline{\mathbf{A}}_{i3}]$$

and using it in (9), and (10) will result in

$$\dot{\mathbf{x}}_1 = \overline{\mathbf{A}}_{11}\mathbf{x}_1 + \mathbf{r} \tag{11}$$

and

$$\mathbf{z} = \overline{\mathbf{A}}_{21}\mathbf{x}_1 \tag{12}$$

where

$$\mathbf{r} = \overline{\mathbf{A}}_{12}\mathbf{y}_1 + \overline{\mathbf{A}}_{13}\mathbf{y}_2 + \mathbf{D}_1\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 + \overline{\mathbf{B}}_1\mathbf{U}$$

and

$$\mathbf{z} = \dot{\mathbf{y}}_1 - \mathbf{D}_2\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 - \overline{\mathbf{A}}_{22}\mathbf{y}_1 - \overline{\mathbf{A}}_{23}\mathbf{y}_2 - \overline{\mathbf{B}}_2\mathbf{U}$$

Since the dynamical system represented in (11), and (12) is driven by totally known input **r**, its state can be estimated by using a conventional Luenberger observer. The dynamics of this observer is given by

$$\dot{\hat{\mathbf{x}}}_1 = \overline{\mathbf{A}}_{11}\hat{\mathbf{x}}_1 + \mathbf{r} + \mathbf{M}(\mathbf{z} - \overline{\mathbf{A}}_{21}\hat{\mathbf{x}}_1) \tag{13}$$

where **M** is the observer's gain. Substituting for **r** and **z** into (13) results in

$$\dot{\hat{\mathbf{x}}}_1 = (\overline{\mathbf{A}}_{11} - \mathbf{M}\overline{\mathbf{A}}_{21})\hat{\mathbf{x}}_1 + (\overline{\mathbf{A}}_{12} - \mathbf{M}\overline{\mathbf{A}}_{22})\mathbf{y}_1 + (\overline{\mathbf{A}}_{13} - \mathbf{M}\overline{\mathbf{A}}_{23})\mathbf{y}_2 + (\overline{\mathbf{B}}_1 - \mathbf{M}\overline{\mathbf{B}}_2)\mathbf{U} + [(\mathbf{D}_1 - \mathbf{M}\mathbf{D}_2)\mathbf{D}_3^{-1}\dot{\mathbf{y}}_2 + \mathbf{M}\dot{\mathbf{y}}_1] \tag{14}$$

Equation (14) contains the derivative of the outputs which is not available for direct measurement. To alleviate this problem a new variable is defined in order to eliminate the need for differentiating the output. To do this let us define a new variable **W** as follow

$$\mathbf{W} = \hat{\mathbf{x}}_1 - [(\mathbf{D}_1 - \mathbf{M}\mathbf{D}_2)\mathbf{D}_3^{-1}\mathbf{y}_2 + \mathbf{M}\mathbf{y}_1] \tag{15}$$

Rewriting (14) in terms of the new variable in (15), will result in

$$\dot{\mathbf{W}} = \mathbf{F}\mathbf{W} + \mathbf{E}\mathbf{Y} + \mathbf{L}\mathbf{U} \tag{16}$$

where

$$\mathbf{H} = (\bar{\mathbf{A}}_{13} - \mathbf{M}\bar{\mathbf{A}}_{23}) + (\bar{\mathbf{A}}_{11} - \mathbf{M}\bar{\mathbf{A}}_{21})(\mathbf{D}_1 - \mathbf{M}\mathbf{D}_2)\mathbf{D}_3^{-1} \tag{17}$$

$$\mathbf{G} = (\bar{\mathbf{A}}_{12} - \mathbf{M}\bar{\mathbf{A}}_{22}) + (\bar{\mathbf{A}}_{11} - \mathbf{M}\bar{\mathbf{A}}_{21})\mathbf{M} \tag{18}$$

$$\mathbf{F} = (\bar{\mathbf{A}}_{11} - \mathbf{M}\bar{\mathbf{A}}_{21}) \tag{19}$$

$$\mathbf{L} = (\bar{\mathbf{B}}_1 - \mathbf{M}\bar{\mathbf{B}}_2) \tag{20}$$

$$\mathbf{E} = [\mathbf{G} \quad \mathbf{H}] \tag{21}$$

The following theorem will finally conclude and summarize the design of the UIO.

Theorem- If the pair $\{\bar{\mathbf{A}}_{21}, \bar{\mathbf{A}}_{11}\}$ is observable, the state of the dynamical system given in (1) can be estimated by using the UIO proposed in (16-21). The estimate of the state is given by

$$\hat{\mathbf{X}} = \begin{pmatrix} \hat{\mathbf{x}}_1 \\ \mathbf{Y} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{0} \end{pmatrix}\mathbf{W} + \begin{pmatrix} \mathbf{N} \\ \mathbf{I} \end{pmatrix}\mathbf{Y} \tag{22}$$

where

$$\mathbf{N} = [\mathbf{M} \quad (\mathbf{D}_1 - \mathbf{M}\mathbf{D}_2)\mathbf{D}_3^{-1}]$$

In addition, all the eigenvalues of **F** can be placed at any desired location.

The proof of the above theorem has been implicitly given in the foregoing discussion.

3) Illustrative Example

In order to illustrate the applicability of the UIO proposed here, consider the following third order dynamical system described by

$$\dot{\mathbf{X}} = \begin{pmatrix} -1.0 & 2.0 & 1.0 \\ -1.0 & 0.0 & 0.0 \\ 0.0 & -1.0 & -1.0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \mathbf{U} + \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \mathbf{v}$$

$$\mathbf{Y} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{X}$$

Since two of the state variable are directly measurable, a first order UIO is required to estimate the unmeasurable state variable. Notice also that the observation matrix **C** is in the desired form described in (2), so no transformation is necessary to get it into the proper form. Preforming the transformation in (8) with the matrix operator **T** in (7) given as

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

will result in

$$\begin{pmatrix} \dot{x}_1 - \dot{y}_2 \\ \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & -1 \end{pmatrix} \mathbf{X} + \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix} \mathbf{U} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \mathbf{V}$$

The eigenvalues of the open loop system are all located in the left hand complex plane and are given by {-1,-0.5±j0.866} therefore an observer gain **M** = -4 was selected so that the closed loop pole of the UIO is placed at -5. Using (16-21) the UIO's dynamic was obtained as

$$\dot{\mathbf{W}} = -5\mathbf{W} + [22 \qquad -4]\,\mathbf{Y} - \mathbf{U}$$

and the state estimate is obtained from

$$\hat{\mathbf{X}} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \mathbf{W} + \begin{pmatrix} -4 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{Y}$$

Figure one shows the response of the system along with the observer to a unit step known input, and a step input of magnitude 5 for the unknown input, with the initial conditions given by $\mathbf{X}^T(0)$ = [1 -1 2]. In Figure two the two (estimated and actual) state trajectories are plotted. From these plots it is obvious that the UIO is performing as desired.

Figure 1-Response of the UIO and the system

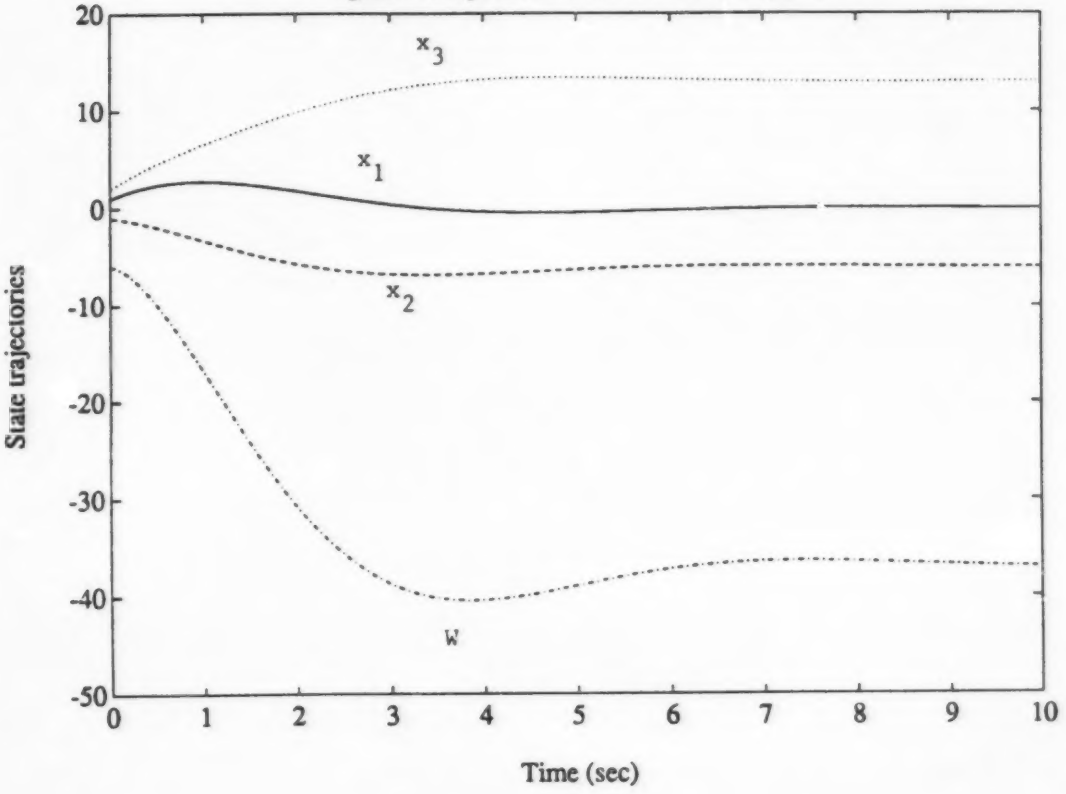
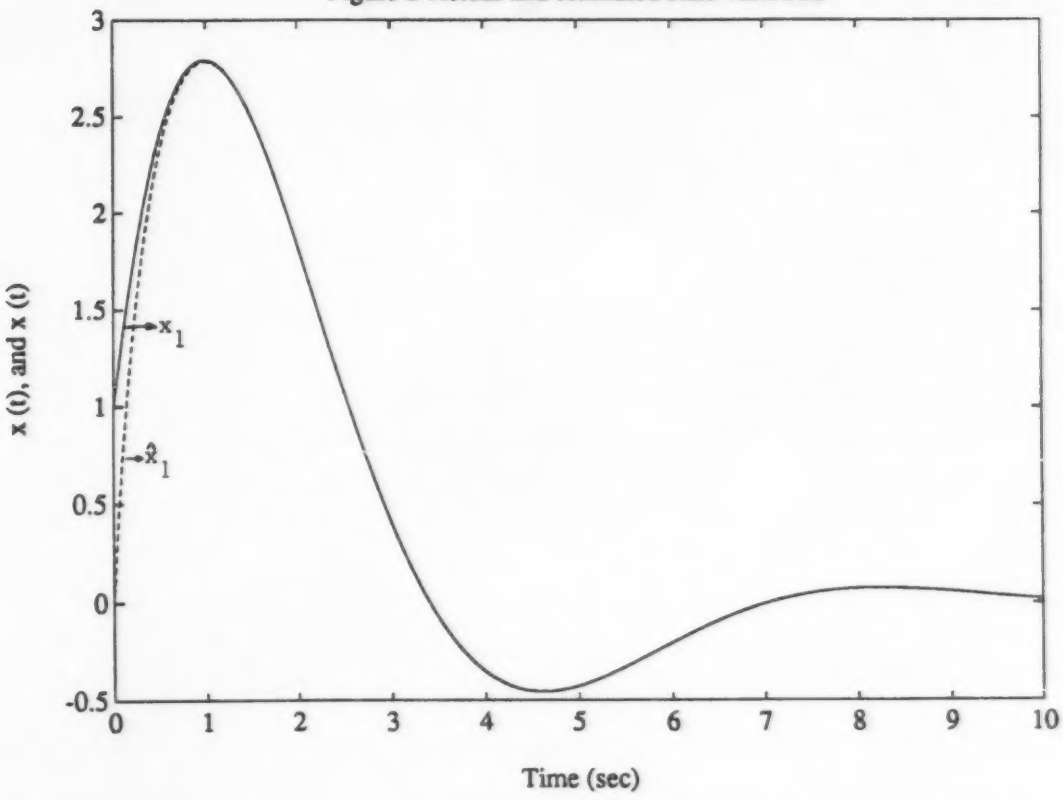


Figure 2-Actual and estimated state variables



4) Conclusion

A new approach for the design of unknown input observers (UIO) capable of estimating the state of a linear dynamical system driven with both known and unknown inputs was presented in this paper. By carefully examining the dynamic systems involved it was possible to transform or redefine the state variables of a dynamical system into a form where it could be partitioned into two interconnected subsystems one of which was directly driven by known inputs only. Therefore, it was possible to use a conventional Luenberger's observe with slight modification for the purpose of estimating the state of the system. As a result, It was also possible to state similar necessary and sufficient conditions to that of a conventional observer for existence of a stable estimator and also arbitrary placement of the eigenvalues of the observer. Finally, it is felt that the design and computational complexities involved in designing UIOs is greatly reduced in the proposed approach.

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